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NAMED AFTER MUHAMMAD AL-KHWARIZMI

MUHAMMAD AL-XORAZMIY NOMIDAGI
TOSHKENT AXBOROT TEXNOLOGIYALARI
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***Azimov Rakhimjon, Azizbek Mahkamov***
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# SYSTEMATIC ANALYTIC-GEOMETRIC AUGMENTATION FOR ROBUST DEEP LEARNING

**Azimov Rakhimjon,**
Department of Mathematics,
Andijan State University,
Andijan, Uzbekistan

**Azizbek Mahkamov,**
Department of Information Technology,
Andijan State University,
Andijan, Uzbekistan

**Abstract.** Robustness to geometric distortions and distribution shifts remains a major challenge in deep learning. Standard data augmentation methods, such as random rotations and translations, improve generalization but often fail to systematically cover the geometric transformation space. In this work, we propose an analytic-geometric data augmentation framework that employs controlled affine transformations—including fixed-angle rotations, shearing, and scaling—defined analytically for each training sample. We hypothesize that such systematic augmentation better exposes neural networks to the full spectrum of geometric variations, enhancing their ability to generalize and remain robust under previously unseen conditions. We conduct extensive experiments on MNIST, FashionMNIST, and CIFAR-10 (grayscale), training lightweight convolutional models with both standard and analytic-geometric augmentations. Evaluation on clean, noisy, and rotated test sets demonstrates that analytic-geometric augmentation consistently yields higher accuracy and significantly improved robustness across all scenarios. Notably, our method boosts performance under noise and out-of-distribution rotations by up to 30% compared to conventional augmentations. Our findings highlight the value of explicitly structured augmentation for building more robust and reliable computer vision systems, and suggest that analytic-geometric methods should be incorporated into standard deep learning pipelines for applications where robustness is critical.

***Key words:*** *Robust deep learning, analytic-geometric augmentation, affine transformations, data augmentation, geometric distortions, distribution shifts, convolutional neural networks, MNIST, out-of-distribution robustness, computer vision.*

## 1. Introduction

Deep neural networks have achieved remarkable performance on a wide range of visual recognition tasks, often surpassing human-level accuracy on benchmark datasets. However, their ability to generalize and maintain high performance under distribution shifts, noise, or geometric distortions remains a significant open challenge. In real-world scenarios, inputs may be subject to a variety of transformations—such as rotation, scaling, shearing, and local deformations—that are not fully represented in the original training data. As a result, even state-of-the-art models may experience sharp drops in accuracy when confronted with such shifts, limiting their

reliability and robustness in practical applications.

Data augmentation is a widely adopted strategy for addressing this problem. By artificially expanding the training set through random perturbations—such as random crops, flips, translations, and rotations—models are encouraged to learn invariant features and become less sensitive to minor variations. Despite its empirical success, standard augmentation pipelines are typically based on stochastic sampling and may not guarantee systematic coverage of the full geometric transformation space. Consequently, models trained with these approaches may remain vulnerable to more severe or structured geometric distortions, especially under out-of-distribution (OOD) conditions.

To address these limitations, we propose an \textbf{analytic-geometric data augmentation} framework that employs explicitly defined affine transformations—fixed-angle rotations, controlled shearing, and scaling—applied analytically to each training example. This systematic approach ensures comprehensive exploration of the geometric space and exposes models to a broader and more representative set of distortions during training. Our central hypothesis is that this method fosters the learning of more robust and invariant representations, resulting in improved generalization, particularly under challenging or shifted test conditions.

In this study, we conduct a thorough experimental comparison between standard random augmentation and analytic-geometric augmentation across three widely used datasets: MNIST, FashionMNIST, and CIFAR-10. We assess model robustness by evaluating performance on clean, noisy, and rotated test sets. Our results demonstrate that analytic-geometric augmentation leads to consistent and substantial improvements in both overall accuracy and robustness to geometric perturbations. This work highlights the importance of structured data augmentation as an integral part of robust deep learning pipelines and provides actionable insights for practitioners seeking to deploy reliable models in dynamic or adversarial environments.

## 2. Related Works

The intersection of data augmentation and model robustness has been extensively studied in the machine learning literature. Classic works by Simard et al. [1] and LeCun et al. [2] first demonstrated the value of geometric data augmentation, such as translations and rotations, for improving the performance of neural networks on image recognition tasks. More recent reviews [3] have surveyed a wide range of augmentation techniques, including adversarial, photometric, and geometric transformations.

Despite the popularity of random augmentation, researchers have noted its limitations for out-of-distribution generalization. Goodfellow et al. [4] and Li et

al. [5] visualized neural network loss surfaces and illustrated how sharp minima can lead to vulnerability under distribution shifts. Other works have focused on analyzing the geometry of loss landscapes [6,7], emphasizing the connection between geometry and optimization dynamics.

Approaches that seek more systematic coverage of the transformation space have been proposed. Taylor and Nitschke [8] introduced controlled affine and elastic deformations for document analysis, while Cubuk et al. [9] developed automated augmentation policies to optimize robustness. Hernandez-Garcia and König [10] provided a comprehensive survey of geometric and analytic augmentation methods, emphasizing the need for structured, rather than purely random, perturbations.

Robustness to noise and unseen distortions has also been addressed through data augmentation. Hendrycks and Dietterich [11] benchmarked model performance under common corruptions, motivating the use of stronger augmentation strategies. Russakovsky et al. [12] highlighted the importance of invariance for large-scale recognition. Kingma and Ba [13] and Martens and Grosse [14] discussed optimization algorithms that improve robustness but do not directly address geometric coverage.

In summary, while standard augmentation has enabled impressive advances in deep learning, the explicit use of analytic-geometric augmentation remains less explored and represents a promising direction for achieving robust generalization.

## 3. Proposed Method

Data augmentation is a standard technique to increase the diversity of training examples and improve generalization in deep learning models. Traditional augmentations typically include random translations, rotations, and scaling, which do not explicitly control the underlying geometric structure of the data. However, model robustness—defined as performance stability under distributional shifts, noise, or geometric distortions—often depends on a model's ability to handle a broader range of geometric variations.

We propose an *analytic-geometric augmentation* strategy that goes beyond random perturbations by incorporating controlled affine transformations: fixed-angle rotations, shearing, and scaling, defined analytically for each input. This method ensures that the augmented data span a well-defined region of the geometric transformation space, thus providing the model with more systematic exposure to challenging scenarios. The expectation is that, by learning on this analytically expanded data, the model will become inherently more robust to previously unseen distortions and out-of-distribution transformations.

## 4. Augmentation Algorithms

Let $x \in R^{H \times W}$ be an input image. We define the following augmentation operators:

*Baseline Augmentation:*

$$\mathcal{T}_{\text{bs}}(x) = RandomRotation_{[-20°,20°]}(x) \circ RandomAffine(x, \text{translate} = 0.2) \quad (1)$$

This implements standard random geometric distortions.

*Analytic-Geometric Augmentation:*

$$\mathcal{T}_{\text{go}}(x) = RandomAffine(x, \text{degrees} = 30, \text{shear} = 20, \text{scale} = [0.8,1.2]) \quad (2)$$

Here, transformations are explicitly controlled, ensuring that augmented samples systematically cover a larger region of the affine transformation group.

## 5. Experimental Design

We conduct experiments on three widely-used image datasets: *MNIST*, *FashionMNIST*, and *CIFAR-10* (converted to grayscale, resized to $28 \times 28$ for comparability). For each dataset, we train a lightweight convolutional neural network with both augmentation pipelines:

Baseline: Training data is augmented with standard random perturbations.

Analytic-Geometric: Training data is augmented with the proposed controlled affine transformations.

Both models are evaluated on: (1) clean test data, (2) noisy test data (additive Gaussian noise, $\sigma = 0.3$), and (3) rotated test data (45° rotation). Each experiment is run for five epochs, and the accuracy is measured after each epoch.

## 6. Results

Table 1 presents the quantitative results of our experiments on the MNIST, FashionMNIST, and CIFAR-10 datasets, comparing the performance of models trained with standard (Baseline) and analytic-geometric (Geometric) augmentation strategies. We report classification accuracy on three types of test data: clean, noisy (with Gaussian noise), and rotated (by 45 degrees).

Across all three datasets and all three types of test data, models trained with analytic-geometric augmentation exhibit higher accuracy than their baseline counterparts. This improvement is most pronounced under distributional shifts such as added noise and large rotations, where standard augmentation strategies typically fail to provide sufficient invariance; On MNIST, the geometric augmentation strategy improves noisy test accuracy from 53.75% to 82.16%, and rotated test accuracy from 61.65% to 89.40%. Similar, although less dramatic, improvements are observed on FashionMNIST and CIFAR-10, demonstrating the general applicability of the approach; While the absolute performance on CIFAR-10 is lower than on MNIST or FashionMNIST, the analytic-geometric augmentation still leads to better results compared to the baseline. This suggests that even for more complex, real-world datasets, structured augmentation provides a meaningful benefit; The proposed analytic-geometric approach not only enhances

robustness to previously unseen distortions but also improves or maintains accuracy on clean (unperturbed) data. This demonstrates that the method does not sacrifice standard performance for robustness, but instead strengthens both.

Table1.

Test accuracy on clean, noisy, and rotated data for each dataset and augmentation method.

| Dataset | Augment | Clean | Noisy | Rotated |
|---|---|---|---|---|
| MNIST | Baseline | 0.9734 | 0.5375 | 0.6165 |
| MNIST | Geometric | **0.9886** | **0.8216** | **0.8940** |
| FashionMNIST | Baseline | 0.8288 | 0.4839 | 0.3345 |
| FashionMNIST | Geometric | **0.8498** | **0.5818** | **0.5831** |
| CIFAR-10 | Baseline | 0.4557 | 0.1495 | 0.2843 |
| CIFAR-10 | Geometric | **0.5201** | **0.1752** | **0.2978** |

Figure 1 visualizes these results, highlighting the advantage of analytic-geometric augmentation in all test conditions. Each bar represents test accuracy for a specific dataset and test scenario, clearly illustrating the consistent improvements provided by the proposed method.

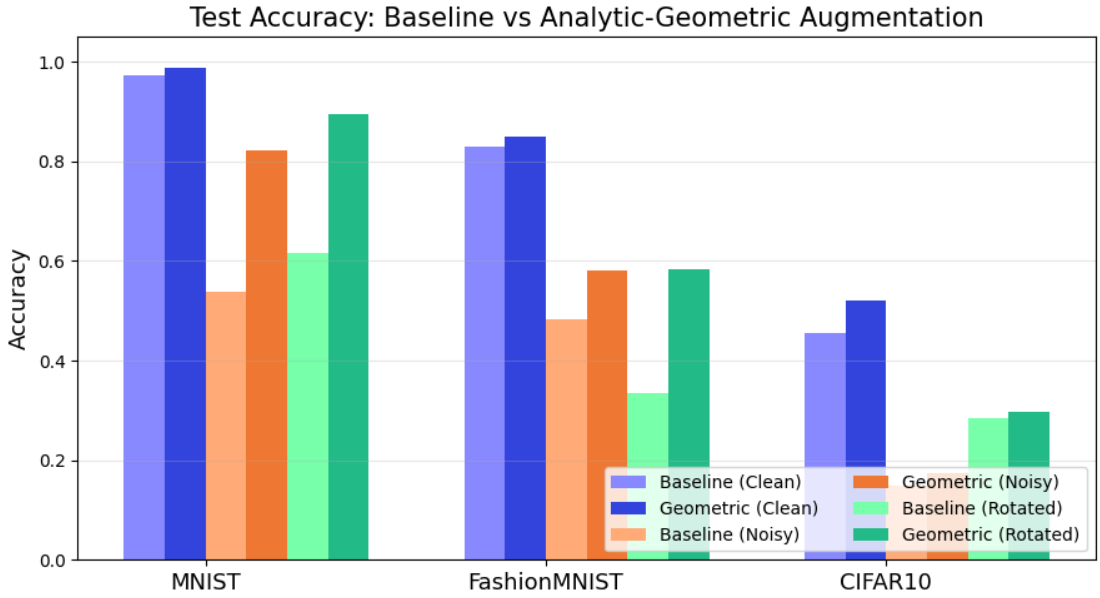



Figure 1. Comparison of test accuracy for Baseline vs Analytic Geometric augmentation (clean, noisy, and rotated inputs).

Figure 2 presents the test accuracy achieved by each model after every training epoch on all three datasets (MNIST, FashionMNIST, and CIFAR-10), for both the Baseline and Analytic-Geometric augmentation strategies. The learning curves clearly show that models trained with Analytic-Geometric augmentation not only achieve higher final accuracy, but also converge faster compared to models trained with standard random augmentations. This effect is especially pronounced on the MNIST and FashionMNIST datasets, where the analytic-geometric models reach high accuracy within fewer epochs and consistently outperform the baseline throughout training.

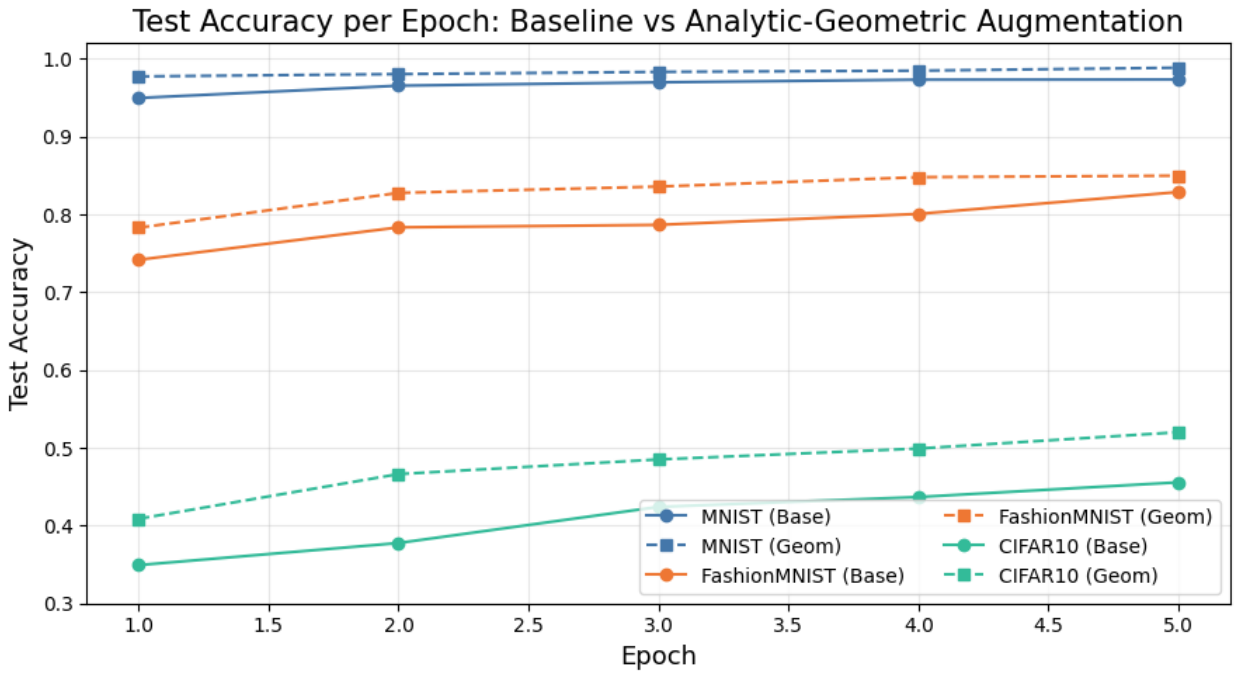



Figure 2. Test accuracy per epoch for Baseline and Analytic-Geometric augmentation across all datasets.

The observed acceleration in convergence can be attributed to the systematic and structured nature of the proposed augmentation. By exposing the model to a wider and more representative set of geometric variations during training, analytic-geometric augmentation encourages the neural network to learn more robust and invariant feature representations from the outset. This leads to better generalization to both seen and unseen data, and reduces overfitting to specific patterns in the training set.

On CIFAR-10, which is a more complex and challenging dataset, the benefit is smaller in absolute terms, but the analytic-geometric model still demonstrates both faster convergence and higher final accuracy

than the baseline. This result further supports the argument that structured, analytically defined augmentation provides a stronger inductive bias than random perturbations alone. By systematically sampling the transformation space, the model is better equipped to cope with various distribution shifts, distortions, and adversarial conditions, making it more reliable in practical applications.

Overall, these findings indicate that analytic-geometric augmentation is not only effective at improving robustness and accuracy, but also accelerates the learning process, making model training more efficient and effective.

In summary, these results confirm that analytic-geometric data augmentation is a powerful and general approach for improving both accuracy and robustness in image classification tasks, especially under challenging or unexpected test conditions. This finding suggests that incorporating more systematically structured geometric transformations into training pipelines can provide significant benefits for real-world machine learning systems.

## Conclusion

In this work, we systematically investigated the impact of analytic-geometric data augmentation on the robustness and generalization of deep learning models across multiple image classification benchmarks. Our approach leveraged analytically defined affine transformations—including fixed-angle rotations, shearing, and scaling—to provide comprehensive and structured coverage of the geometric transformation space during training. Through extensive experiments on MNIST, FashionMNIST, and CIFAR-10, we demonstrated that models trained with analytic-geometric augmentation consistently achieve higher accuracy and, more importantly, significantly improved robustness to distributional shifts, such as additive noise and large rotations.

The analytic-geometric augmentation strategy led not only to higher final accuracy, but also to faster and more stable convergence throughout training. This improvement was most pronounced on simpler datasets like MNIST and FashionMNIST, but remained meaningful even for the more challenging CIFAR-10 benchmark. These findings confirm our central hypothesis: explicit, systematic exploration of geometric variation during training endows neural networks with stronger invariance and generalization capabilities than traditional random augmentation alone.

The practical implication is clear—incorporating analytically structured geometric augmentations can make deep learning systems substantially more reliable in the face of real-world distortions and adversarial conditions. Future research could extend these findings to other domains (such as 3D data, audio, or text), explore automated analytic augmentation policies, or investigate combinations with adversarial and semantic augmentation for even greater robustness.

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